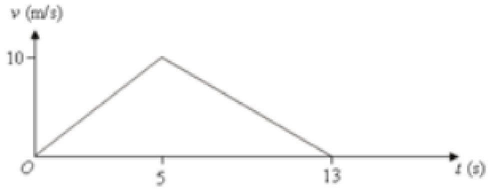
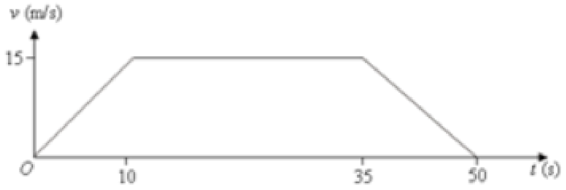
1 Draw the velocity–time graph.



$$\begin{aligned}\text{Distance travelled} &= \text{area under graph}\\ &= \frac{1}{2} \times 10 \times 13\\ &= 65 \text{ m}\end{aligned}$$

2 Draw the velocity–time graph.



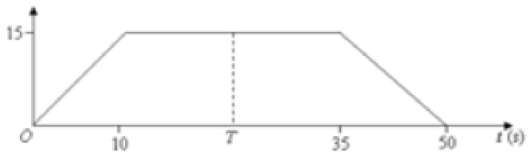
a The area can be calculated using the trapezium formula, or as the sum of two triangles and a rectangle.

$$\begin{aligned}A &= \frac{1}{2}(a+b)h\\ &= \frac{1}{2} \times (25+50) \times 15\\ &= 562.5 \text{ m}\end{aligned}$$

b

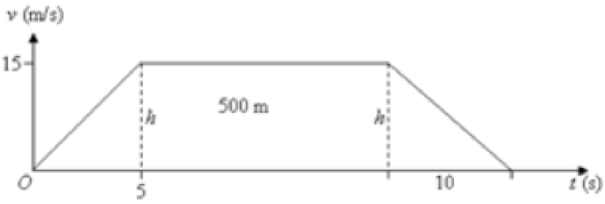
$$\begin{aligned}A &= \frac{1}{2}(a+b)h\\ &= \frac{1}{2} \times (25+35) \times 15\\ &= 450 \text{ m}\end{aligned}$$

c Let the halfway point be at time T as below.



$$\begin{aligned}\frac{1}{2} \times 10 \times 15 + 15(T-10) &= \frac{562.5}{2}\\ 75 + 15T - 150 &= 281.25\\ 15T &= 356.25\\ T &= 23.75 \text{ s}\end{aligned}$$

3

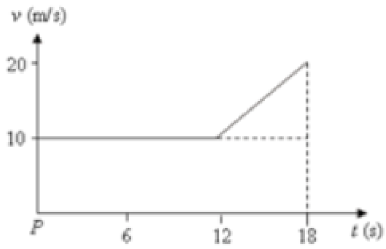


Since the total distance travelled is 1 km or 1000 m, the combined areas of the two triangles will equal a distance of 500 m.

$$\frac{1}{2} \times 5 \times h + \frac{1}{2} \times 10 \times h = 500$$
$$5h + 10h = 1000$$
$$15h = 1000$$
$$h = \frac{1000}{15}$$
$$= 66\frac{2}{3}$$

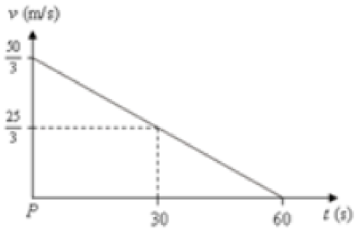
Maximum speed $= 66\frac{2}{3}$ m/s

4 $36 \text{ km/h} = 36 \div 3.6$
$= 10 \text{ m/s}.$
$72 \text{ km/h} = 20 \text{ m/s}.$



Distance $= A = 18 \times 10 + \frac{1}{2} \times 6 \times 10$
$= 210 \text{ m}$

5 $60 \text{ km/h} = 60 \div 3.6$
$= \frac{50}{3} \text{ m/s}$



a Distance $= A = \frac{1}{2} \times 60 \times \frac{50}{3}$
$= 500 \text{ m}$

b Distance $= A = \frac{1}{2} \times \left(\frac{50}{3} + \frac{25}{3}\right) \times 30$
$= 375 \text{ m}$

c Let the required time be T s.

It is easier to work with the triangle on the right.

This triangle will have area

$$= 500 \div 2$$
$$= 250$$

Its base $= (60 - T)$

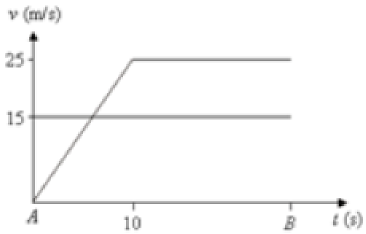
The sloping line has gradient

$$= -\frac{50}{3} \div 60$$
$$= -\frac{50}{180} = \frac{5}{18}$$

$\therefore$ the triangle's height $= \frac{5}{18}(60 - T)$

$$\frac{1}{2} \times (60 - T) \times \frac{5}{18}(60 - T) = 250$$
$$\frac{5}{36}(60 - T)^2 = 250$$
$$(60 - T)^2 = 250 \times \frac{36}{5}$$
$$= 1800$$
$$60 - T = \sqrt{1800}$$
$$\approx 42.43$$
$$T \approx 17.57 \text{ s}$$

6 Let the common time be T s and the distance x m.

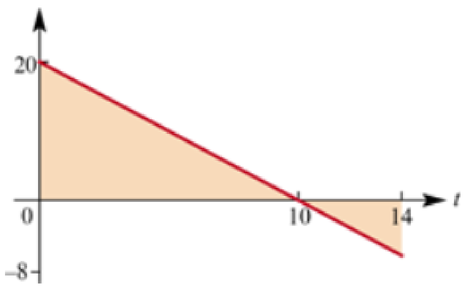


a For the first car, $x = 15t$
For the second car,

$$x = \frac{1}{2} \times 10 \times 25 + 25(t - 10)$$
$$= 125 + 25t - 250$$
$$= 25t - 125$$
$$= 15t$$
$$10t = 125$$
$$t = 12.5 \text{ s}$$

b $x = 15t$
$$= 15 \times 12.5$$
$$= 187.5 \text{ m}$$

7 **a**



b The particle moves to the right for the first 10 seconds. Its position at time t is given by
$s = 20t - t^2$
It slows for the first ten seconds. At time $t = 10$, it is 100 m to the right of its starting point. It then heads to the right for 4 seconds. When $t = 14$ it is 84 m from its starting point.

c Total distance travelled $= 100 + 16 = 116$ m.

d It is 84 m to the right of its starting point.

8 a For the first ten seconds of motion

$$\text{acceleration} = \frac{10 - 0}{10 - 0} = 1 \text{ m/s}^2$$

b From $t = 20$ to $t = 30$ the

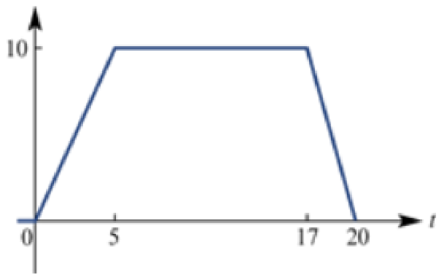
$$\text{acceleration} = \frac{-15 - 10}{30 - 20} = -\frac{5}{2} \text{ m/s}^2$$

c The equation of the line through $(20, 10)$ and $(30, -15)$ is $v - 10 = -\frac{5}{2}(t - 20)$ which can be written as
$v = -\frac{5}{2}t + 60$.
When $v = 0, t = 24$

$$\begin{aligned} \text{Distance travelled in the first 24 s} &= 5(10 + 24) \\ &= 170\text{m} \\ \text{Distance travelled in next 6 s} &= 3 \times 15 \\ &= 45\text{m} \\ \therefore \text{total distance} &= 45 + 170 \\ &= 215\text{m} \end{aligned}$$

d Displacement $= 170 - 45 = 125$ m to the right of its starting point.

9 a



Let $(T, 20)$ be the point at which the constant acceleration ends. The motion ends at $(20, 0)$.
Considering the area of the trapezium:
$5(20 + (T - 5)) = 160$
$\therefore T - 15 = 32$
$\therefore T = 17$

b $\text{acceleration} = \frac{10}{17 - 20} = -\frac{10}{3} \text{ m/s}^2$

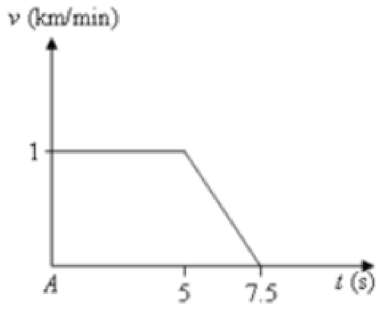
10 Convert the speeds to km/min.

$$60 \text{ km/h} = 1 \text{ km/min}$$

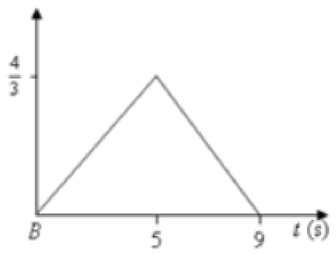
$$80 \text{ km/h} = \frac{4}{3} \text{ km/min}$$

Treat each train separately.

The first train:



The second train:

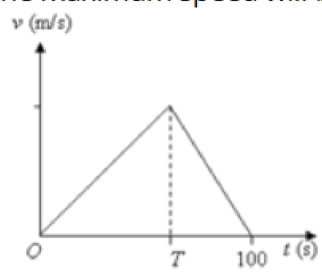


First train distance
$$= 5 \times 1 + \frac{1}{2} \times 2.5 \times 1$$
$$= 6.25 \text{ km}$$

$$\text{Second train distance} = \frac{1}{2} \times 9 \times \frac{4}{3}$$
$$= 6 \text{ km}$$

Since the trains have together travelled less than 14 km, they will not crash.

11a The maximum speed will be the height of the triangle.



$$\frac{1}{2} \times 100 \times h = 800$$
$$50h = 800$$
$$h = 16$$

$$\text{Maximum speed} = 16 \text{ m/s}$$
$$= 16 \times 3.6$$
$$= 57.6 \text{ km/h}$$

b The slope of the deceleration is twice as steep as the slope of the acceleration.
Since the heights are equal, the acceleration run will be twice as long as the deceleration run.

$$T = \frac{2}{3} \times 100$$
$$= 66\frac{2}{3} \text{ s}$$
$$= 1 \text{ min } 6\frac{2}{3} \text{ seconds}$$

c Taking the acceleration section,

$$\text{the gradient} = a = 16 \div 66\frac{2}{3}$$
$$= \frac{48}{200}$$
$$= 0.24\ \text{m/s}^2$$